MEASURING THE TEMPERATURE OF THE SUN

SYNOPSIS: In this lab you will measure the solar flux, the amount of energy per unit area per unit time that reaches the Earth from the Sun. From this you will calculate the temperature of the surface of the Sun.

EQUIPMENT: Insulated cup, water, ink, plastic wrap, thermometer, watch, meter stick, gram scale, graph paper, calculator.

Review: units (p. 11), temperature (p. 13), energy & power (p. 13), trig (p.19)

Introduction

The quantity of energy delivered by sunlight, per unit of area and per unit of time, is called the **energy flux** *F* (in units of *joules per square meter per second*). If we multiply the *flux F* by the *area A* onto which the light falls, and also multiply by the *duration of time   t* that the area is exposed to the light, we obtain the total amount of energy that has been delivered by the Sun:

$$\text{Energy delivered by Sun} = F \cdot A \cdot \ t \ .$$

The *temperature increase   T* resulting from the sunshine depends upon two properties of the absorbing material: its **heat capacity** *C*, and its total mass *M*. The more mass there is to be heated, the less the temperature will rise for a given amount of energy. The heat capacity states how much energy is needed to raise the temperature of a kilogram of the material by one degree; for example, the heat capacity of water is

$$C_{\text{water}} = 4{,}186 \ \frac{\text{joules}}{\text{kg} \cdot \ {}^{\text{o}}\text{C}} \quad ,$$

meaning that it takes 4,186 joules of energy to raise the temperature of one kilogram of water by one degree Celsius.

The amount of energy needed to produce a temperature increase of   *T* in a material of mass *M* and heat capacity *C* is then

$$\text{Energy absorbed from the Sun} = C \cdot M \cdot \ T \ .$$

If all of the energy delivered by sunlight is completely absorbed, the two quantities above are equal:

$$\text{Energy delivered by Sun} \ = \ \text{Energy absorbed from Sun}$$

$$F \cdot A \cdot \ t \ = \ \ C \cdot M \cdot \ T$$

The change in temperature of the material   T is then:

$$T = \frac{F \ A \ t}{C \ M}$$

Note that the increase in temperature is *directly* proportional to the rate *F* at which energy arrives, the size of the energy-absorbing area *A*, and the duration of exposure   *t* to the energy; the

temperature increase is *inversely* proportional to C, the ability of the material to store energy internally, and to the amount of material M that must be heated.

Of course, we don't need to calculate how the temperature of an object exposed to sunlight will increase; all we need is a thermometer. But we *can* rearrange the equation to find out how to measure the really difficult quantity: F, the energy per unit area per unit time arriving from the Sun:

$$ F = \frac{C\ M}{A}\ \frac{T}{t}\ . \qquad (1) $$

Part I. Gathering Data

In this experiment, you will place a container of water out in the sunshine, and use a thermometer to measure the increase in the water temperature   T that occurs over a time interval   t.   You already know the heat capacity of the water, C_{water}. A scale will be used to determine the water's mass M, and a meter stick can be used to determine the surface area A of the water.   These bits of information can then be combined in equation (1) to yield the solar flux F, the number of joules of energy falling each second onto each square meter of the illuminated surface of the Earth.

Prepare your experimental apparatus:

I.1   Measure the inside diameter D of the top of the insulated cup in meters, and calculate its surface area A_{cup} in square meters:

$$ A_{cup}\ =\ \frac{D^2}{4}\ . \qquad (2) $$

The water you use should start out a few degrees cooler than the outside (ambient) air temperature, since the temperature increase will be the most accurate when the air and water temperatures are approximately the same.

I.2   Measure the outside air temperature with your thermometer and measure the temperature of the water. If the water is too warm, chill it with ice cubes or use water pre-chilled by your teaching assistant.

I.3   Weigh the insulated empty container and write down its mass (in kilograms). Fill the cup roughly three-fourths full with water, and add about three drops of ink to the water to help it absorb energy more efficiently.

I.4   Weigh the container again, and calculate the mass of the water M_{water} in kilograms.

Cover the top of the container with plastic wrap to help reduce heat losses from evaporation, and poke the thermometer through the top. Carry the works outside to a sunny, level area protected from wind.

You are now ready to monitor how the incident solar energy affects the temperature of the water.
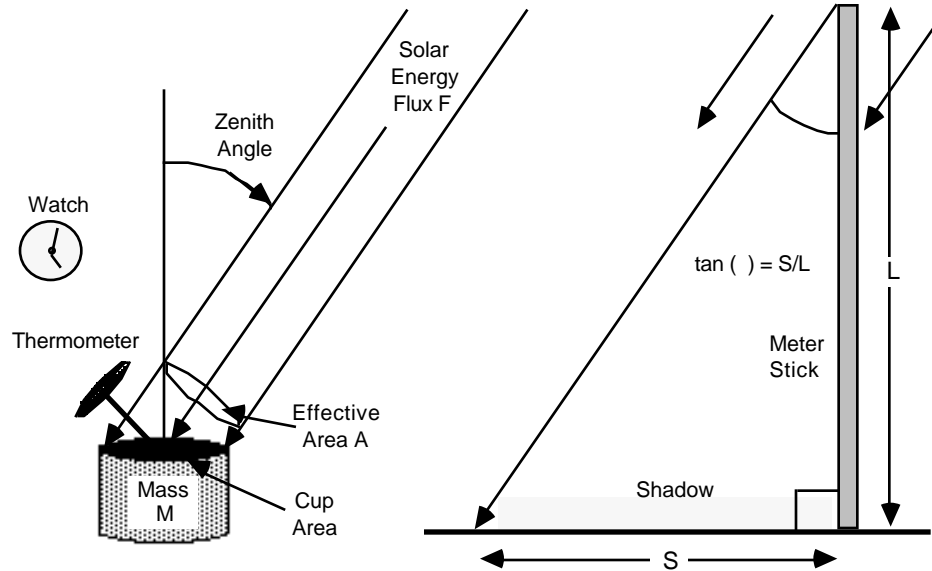
I.5   As the sunlight warms the water, keep a running record of the temperature T of the water versus elapsed time t.   Make a temperature reading once every minute.   Between readings, it is good practice to gently stir the water with the tip of the thermometer to keep the temperature uniform. Continue to take measurements until the water is a few degrees warmer than the ambient temperature; you will need about one-half hour of continuously clear skies.

I.6   You will also need to know the **zenith angle**   of the Sun (how many degrees it is from directly overhead). The angle can be determined by measuring the shadow cast by the meter stick L: hold the meter stick vertically and mark where the tip of its shadow falls on the ground; then use the stick itself to measure the length S of its shadow.   The tangent of the zenith angle will be

$$\tan(\ ) = \frac{S}{L} \ . \tag{3a}$$

Note: If you've already measured the solar altitude for the Season's lab, the zenith angle is simply:

$$\text{Zenith Angle} \ = 90° - (\text{solar altitude}) \tag{3b}$$

You're through gathering data. Carry everything back inside.

Part II. Calculating the Solar Energy Flux

If the Sun had been directly overhead (which is not possible from Boulder's latitude), the energy-absorbing area of the water would equal the area of the cup A_{cup}. However, the oblique angle of the sunlight reduced the effective area A by an amount equal to the cosine of the solar zenith angle:

$$A = A_{cup} \cdot \cos(\ ) \tag{4}$$

II.1   Calculate the zenith angle   from equation (3a) or (3b), and calculate the effective surface area A using equation (4).

II.2   On the graph paper at the back of this exercise, graph your temperature measurements in
         degrees Celsius against the elapsed time in seconds, as shown below. (Don't forget to
   convert minutes into seconds!)
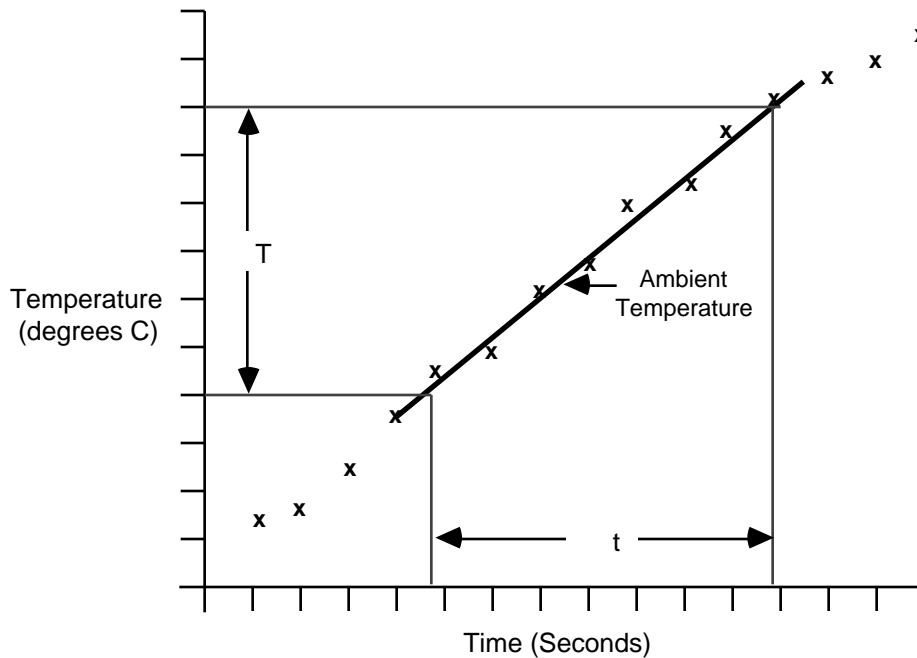
During the period when the water temperature was close to the ambient temperature, the graph
should approximate a straight line, indicating a uniform rate of temperature increase.

II.3   Draw a best-fit straight line through this portion of the graph. Determine the slope of this
         line: find the change in temperature T that occurs over the time interval t.

II.4   Use your experimentally-determined values for the quantitiies on the right-hand side of
         equation (1) to calculate the solar flux F. If you have properly kept track of the units in
         each of your parameters, the solar flux will have units of joules per square meter per
         second (joules/(m^2 · s)).

The **watt** is a unit of *power*, or energy per unit of time. One watt is equal to 1 joule per second, so
that your measurement of the solar flux in "joules/(m^2 · s)" can also be written as "watts/m^2". The
meaning of your measurement of the solar flux is then easier to visualize: it is the power in watts
delivered by the Sun to each square meter of the Earth's surface.

The **solar constant** E is the name given to the solar energy flux arriving just outside the Earth's
atmosphere. A portion of that energy is absorbed by the atmosphere (after all, the air is heated by
sunlight just like the water in the cup!), so your measurement of F will be somewhat smaller than
the value of E. The amount of energy loss in the atmosphere depends upon the zenith angle   of
the Sun and the amount of water vapor in the atmosphere.

II.5   Use the chart below to estimate what fraction of the incident solar radiation actually
          reached the Earth's surface during your experimental measurements. Estimate whether
          the sky was extremely clear, average, or hazy; then use the corresponding curve and your
          measured solar zenith angle to determine the **atmospheric transmission** *X*.
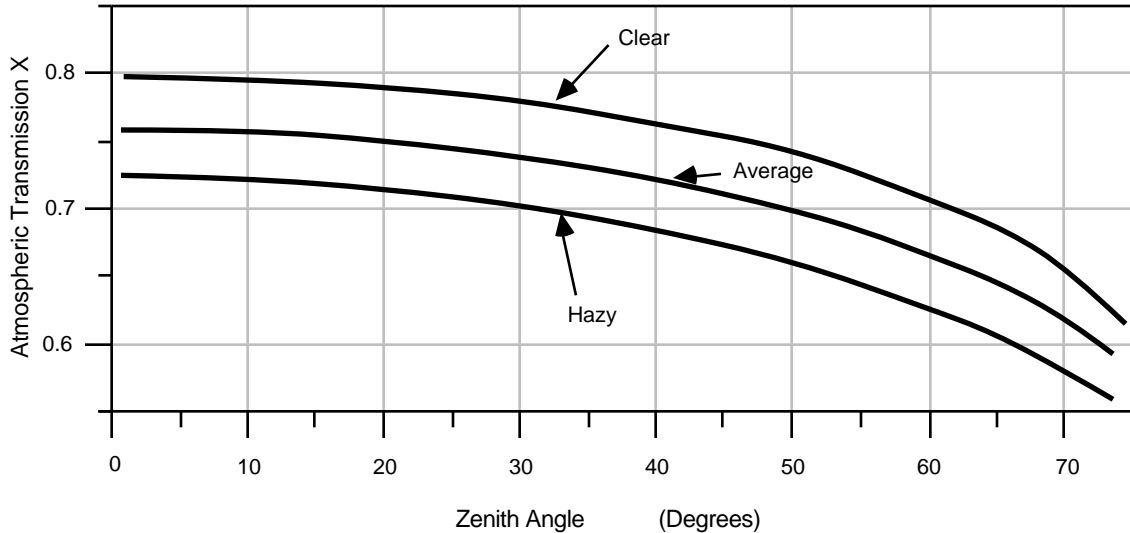
II.6   Calculate the solar constant *E* by dividing your measurement of the flux *F* below the
          atmosphere by the atmospheric transmission *X*:

$$E = \frac{F}{X} \tag{5}$$

II.7   Compare your calculated value for the solar constant with the generally accepted value of
          1,388 watts/m^2 (that's equivalent to nearly fourteen 100-watt light bulbs shining onto an
          area 3 feet by 3 feet!). If your value is significantly different, suggest possible sources of
   error.

Part III. The Temperature at the Surface of the Sun

Your calculated value of the solar constant *E* measured the number of watts striking *one* square
meter of surface at a distance of one astronomical unit (1.5×10^{11} m) from the Sun.   If you
multiply the energy hitting one square meter of the Earth by the total number of square meters on a
sphere of radius 1 AU (reminder: the area of a sphere of radius R = 4 R^2), you obtain the
luminosity, or total power output in watts emitted by the Sun!

III.1   Calculate the luminosity of the Sun in watts:

$$\text{Luminosity} = E \times 4 \; (1.5 \times 10^{11} \text{ m})^2 \quad . \tag{6}$$

          Compare your measurement of the total power output from the Sun with the value given
          in your textbook.
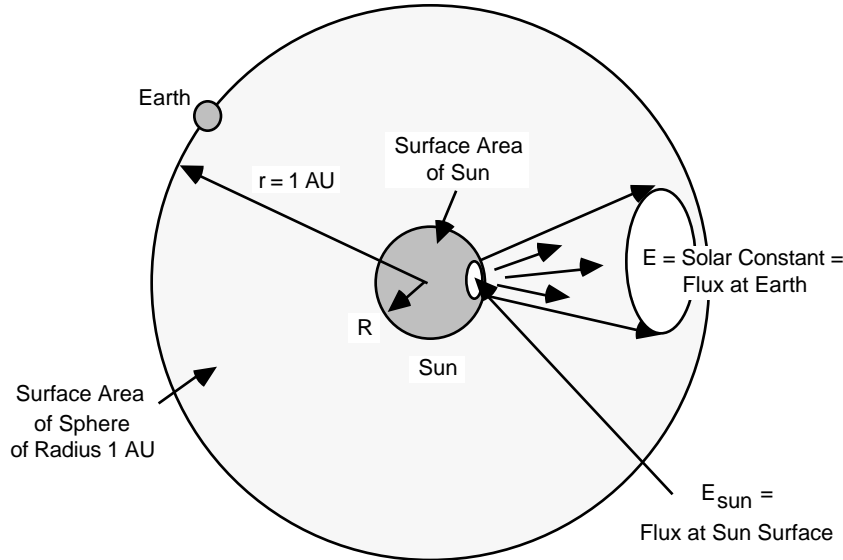
Now that you know how much *total* energy the Sun puts out, you can also determine how much energy E_{sun} is emitted by *each* square meter of the solar photosphere! Since the radius of the Sun is about 7.0×10^8 m, we divide the total energy output by its surface area:

$$E_{sun} = \frac{\text{Luminosity}}{4 \ (7.0 \times 10^8 \text{ m})^2} \ . \hspace{2cm} (7)$$

III.2   Calculate the power per unit area E_{sun}, (watts/m^2) leaving the Sun's surface.

III.3   How many 100-watt lightbulbs would be required to equal the energy output from *each* square meter of the solar photosphere?

The Sun radiates energy according to the Stefan-Boltzmann law: the power radiated per unit area (E_{sun}) is proportional to the fourth power of its absolute temperature T measured in degrees Kelvin:
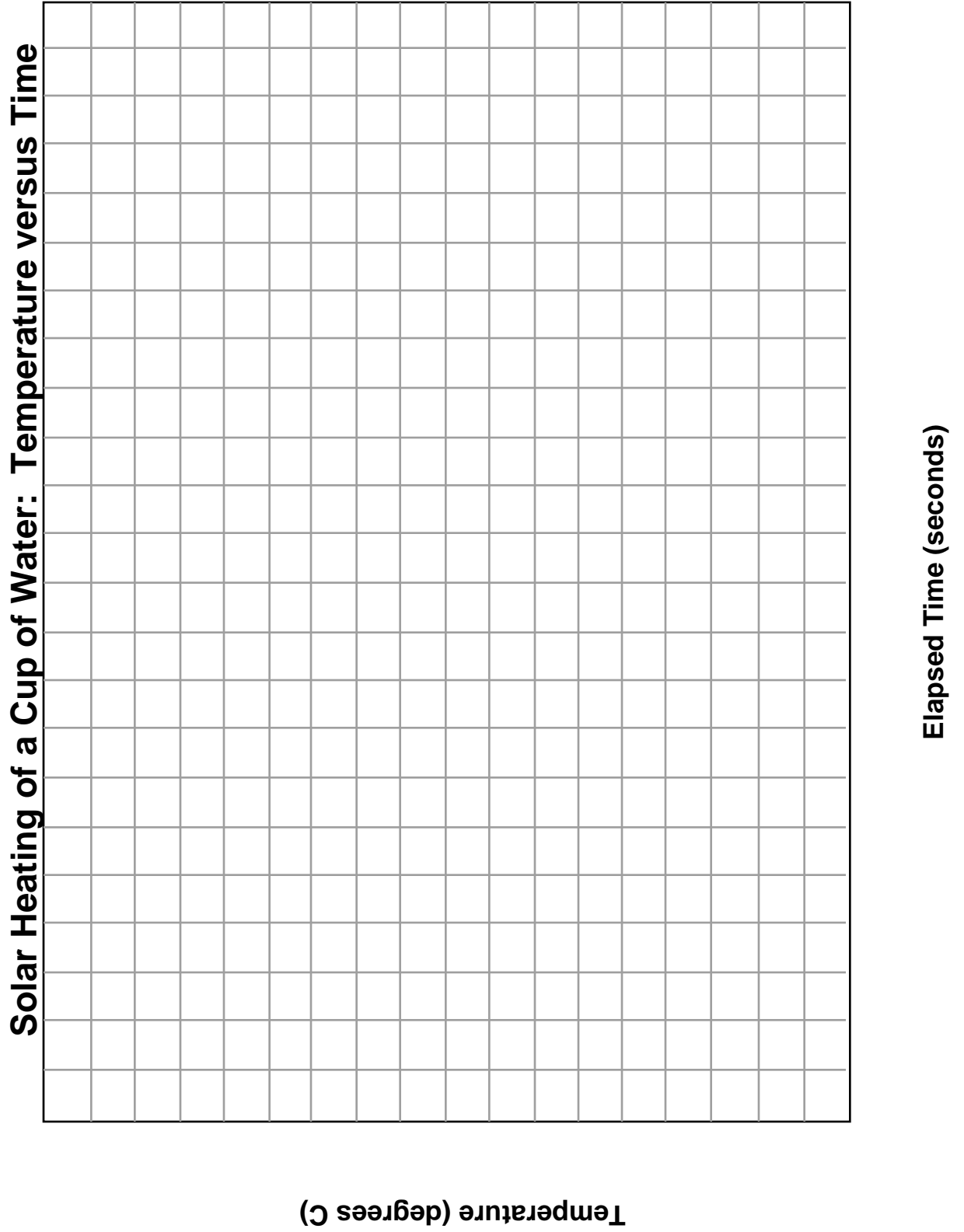
$$E_{sun} = \ T^4$$

where   (called the Stefan-Boltzmann constant) is known from experiment to be 5.69×10^{-8} watts/(m^2 K^4).
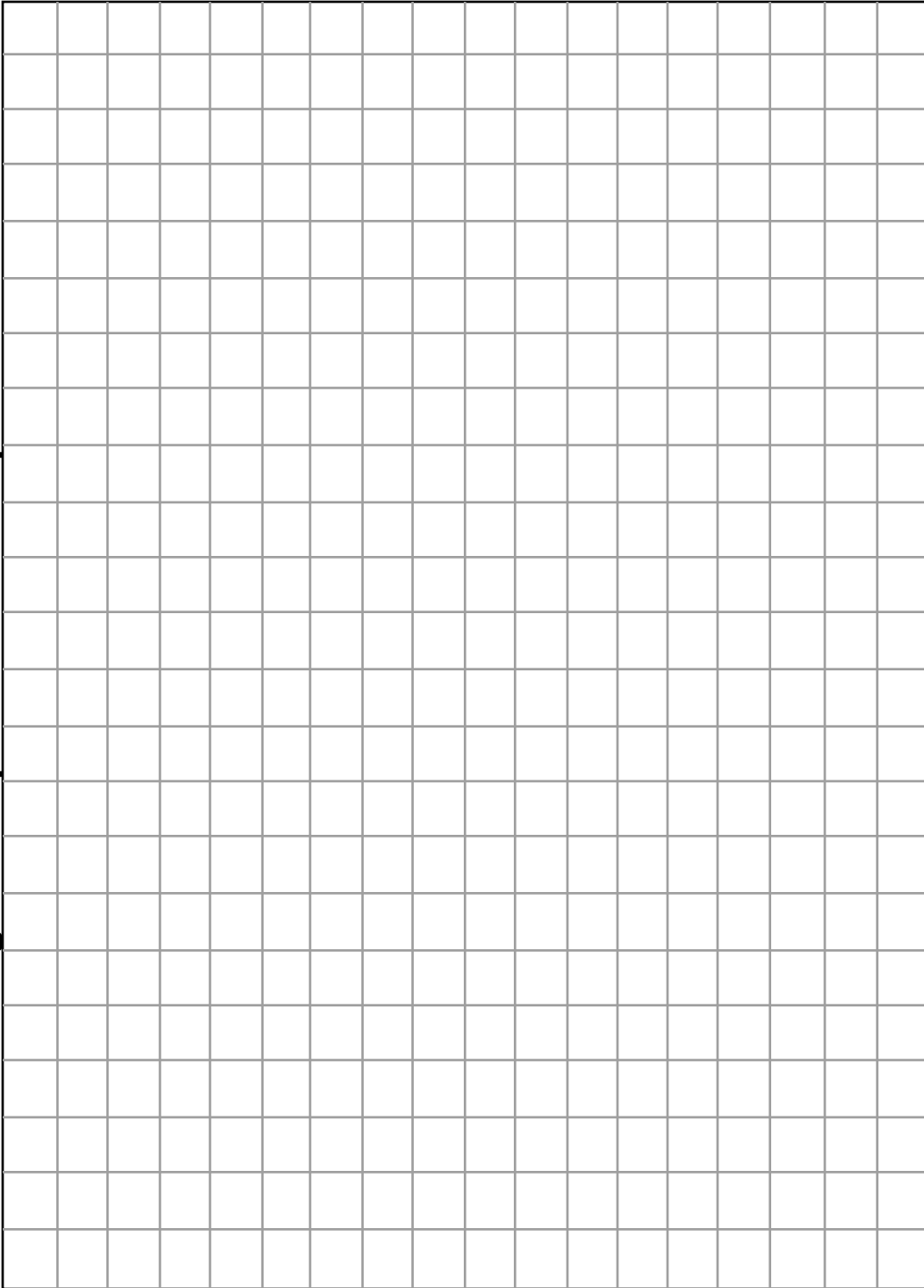
III.4   Calculate the temperature of the Sun's photosphere in degrees Kelvin:

$$T = (E_{sun}/ \ )^{1/4} \hspace{2cm} (8)$$

Does your answer make sense? Compare your measured value with the accepted average solar photosphere temperature of 5800 K.

Solar Heating of a Cup of Water: Temperature versus Time

Elapsed Time (seconds)

Temperature (degrees C)

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