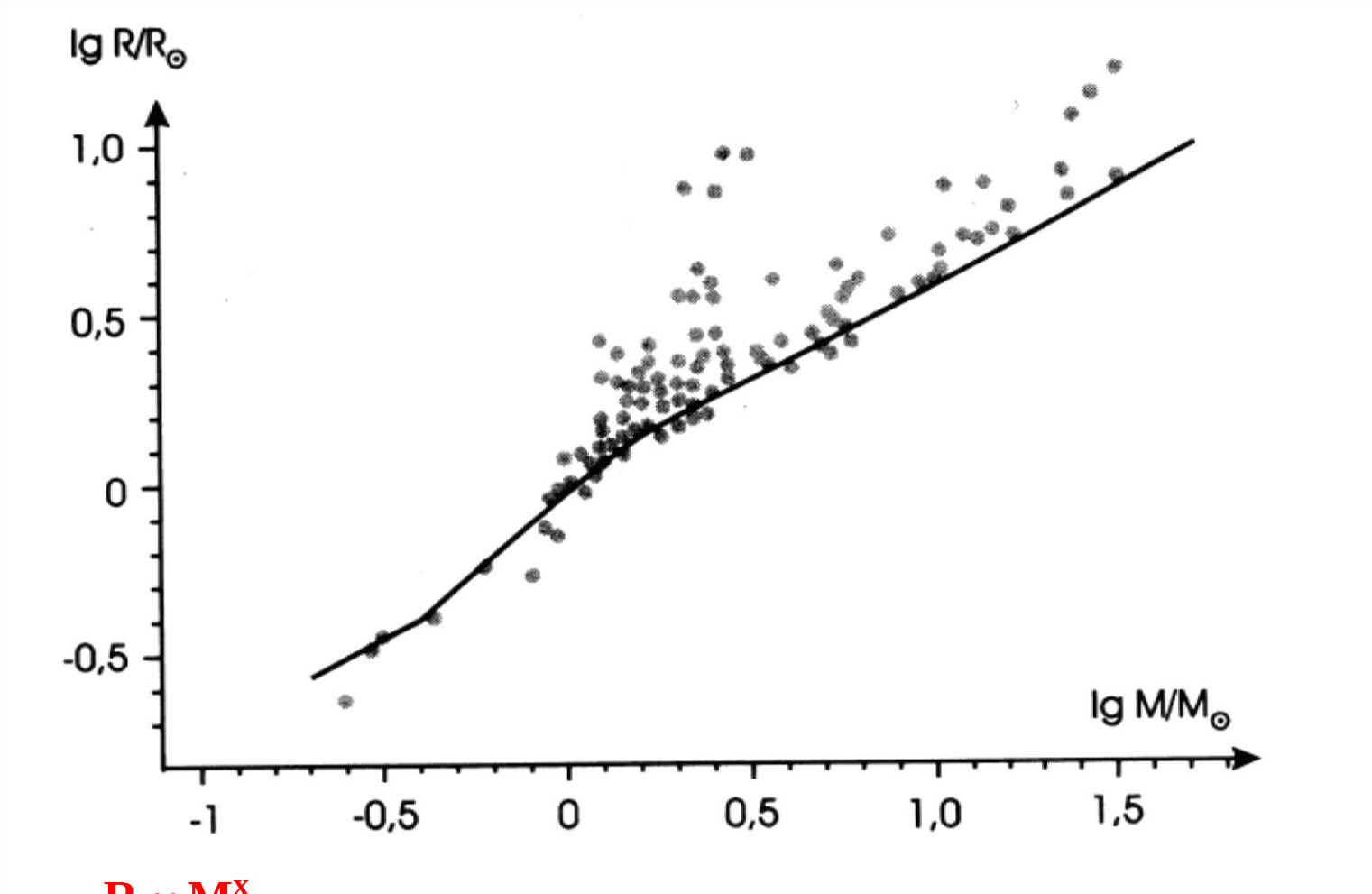
PROSTE MODELE GWIAZDOWE

- **modele ZAMS**
- **politropy**
- **modele homologiczne**

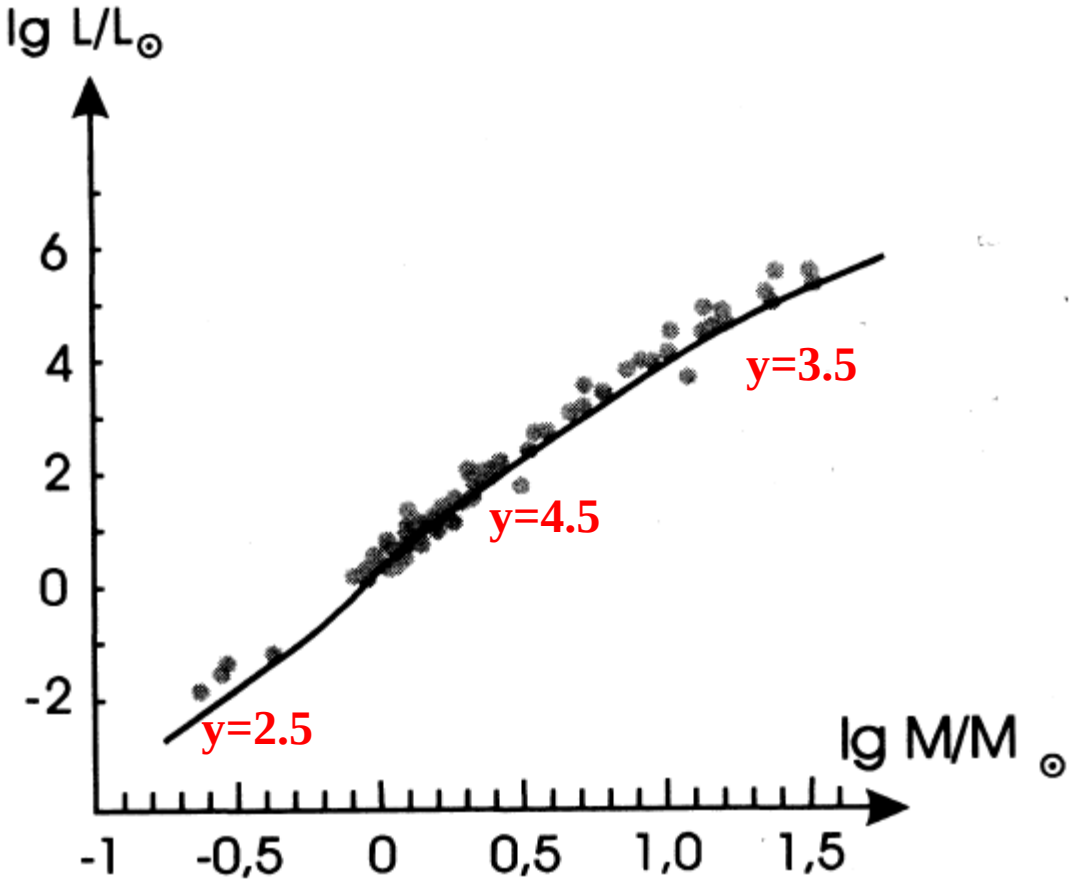


$R \sim M^x$

x=0.56 – górny przedział mas

x=0.79 – dolny przedział mas

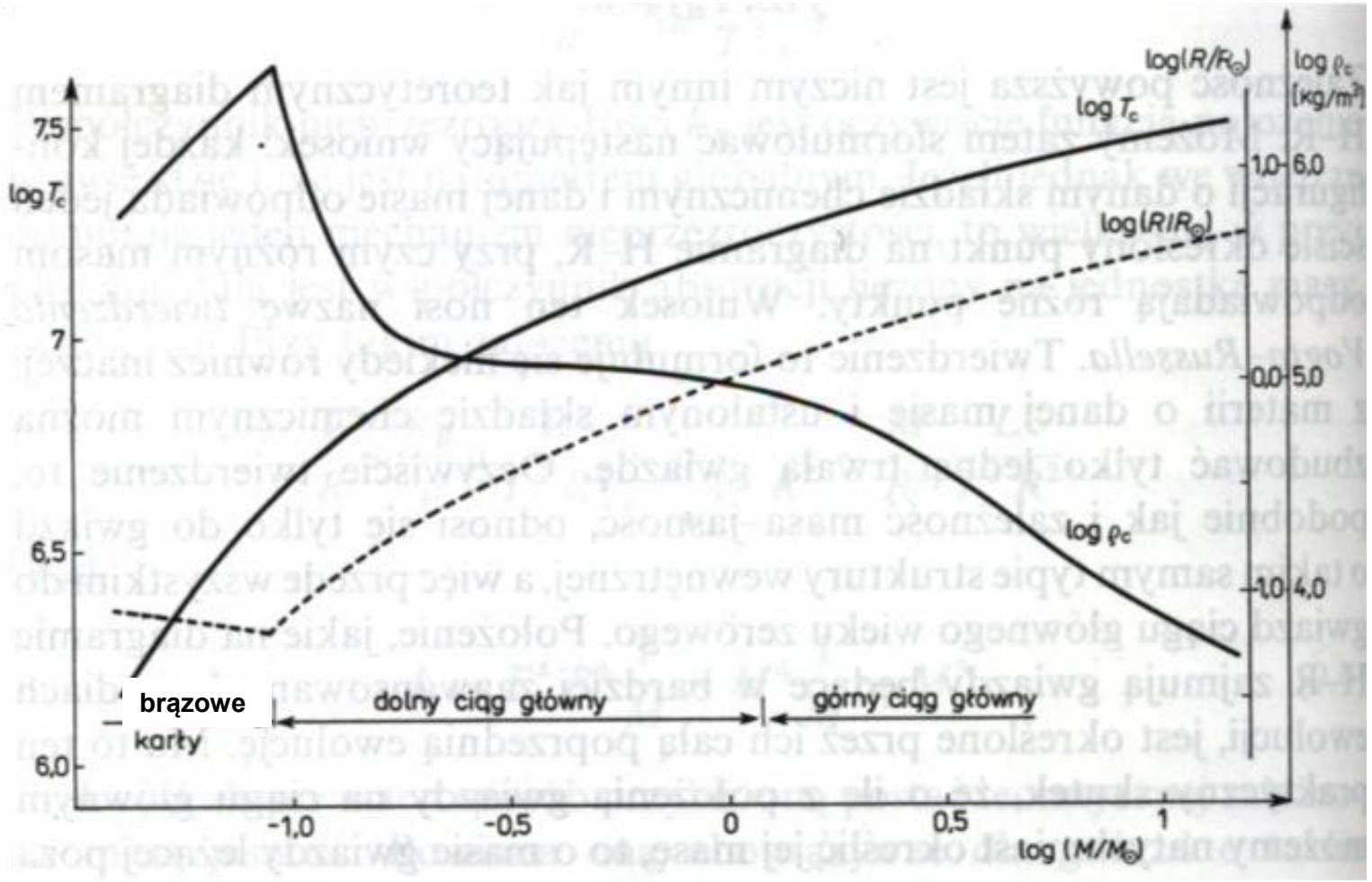
Kippenhahn, Weigert & Weiss 2012



$L \sim M^y$

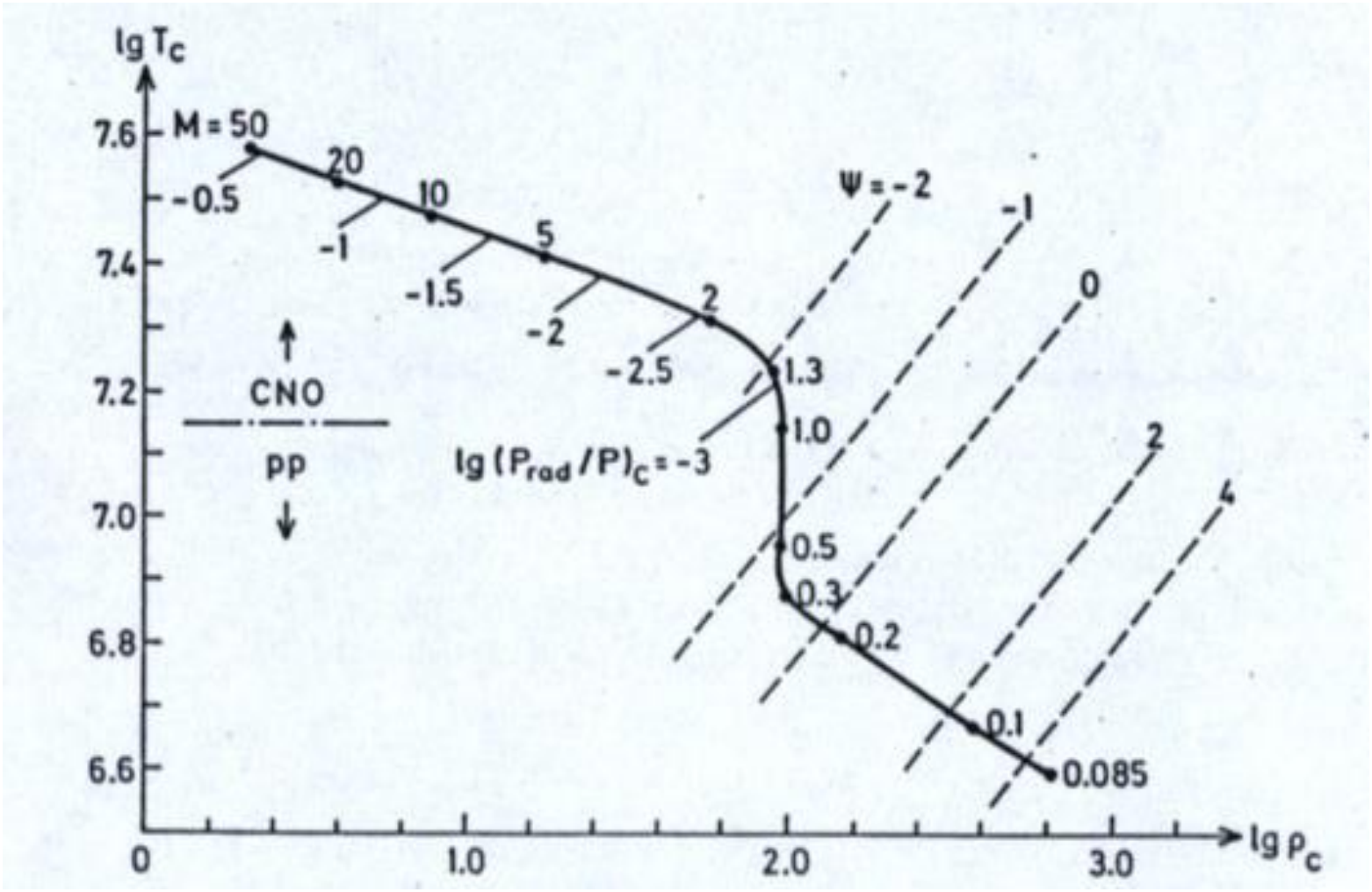
y=2.5 – 4.5

Kippenhahn, Weigert & Weiss 2012


log T_c
7,5
7
6,5
6,0
log $(R/R_\odot)$
log ρ_c (kg/m^3)
log T_c
log $(R/R_\odot)$
log ρ_c
1,0
0,0
-1,0
6,0
5,0
4,0
brązowe karły
dolny ciąg główny
górny ciąg główny
-1,0
-0,5
0
0,5
1
log $(M/M_\odot)$

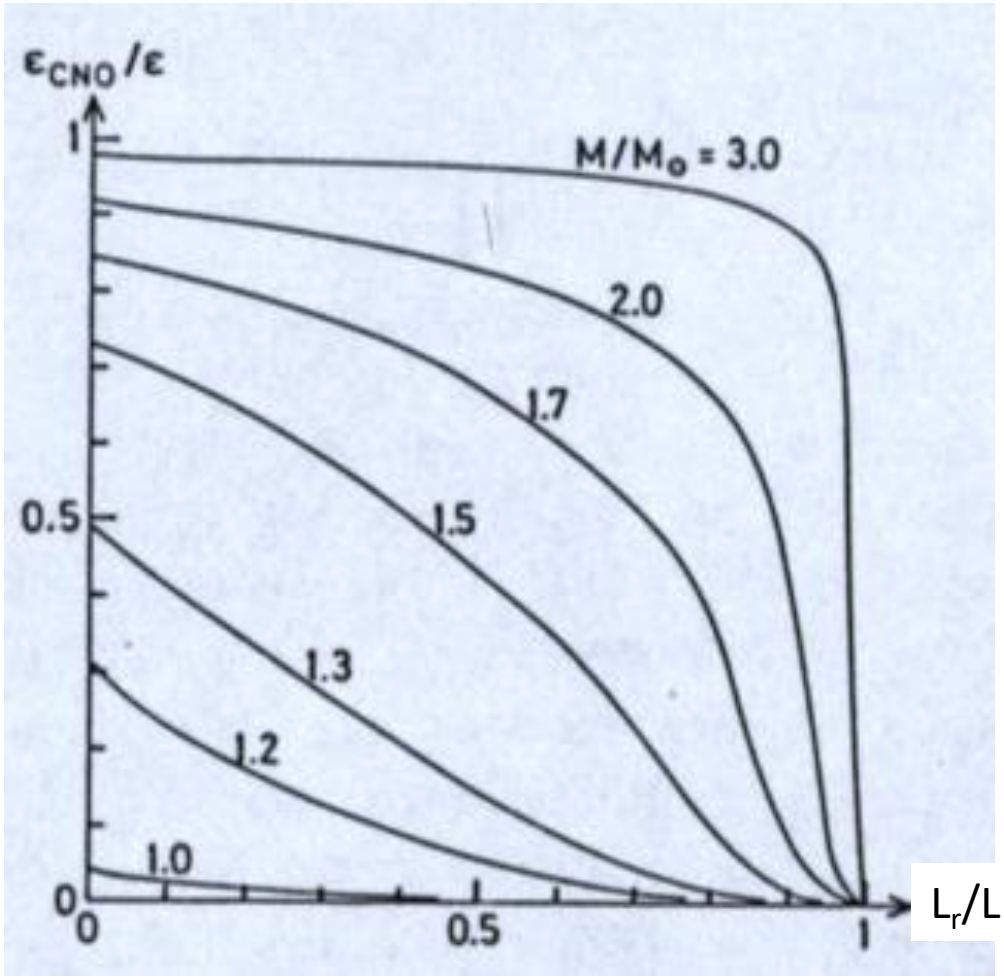
Kubiak 1994

Zależność $\log T_c$ vs. $\log \rho$ dla modeli ZAMS



Kippenhahn & Weigert 1990

Modele ZAMS. Wkład cyklu CNO do całkowitego tempa produkcji energii w funkcji lokalnej jasności.



Kippenhahn & Weigert 1990

Modele ZAMS

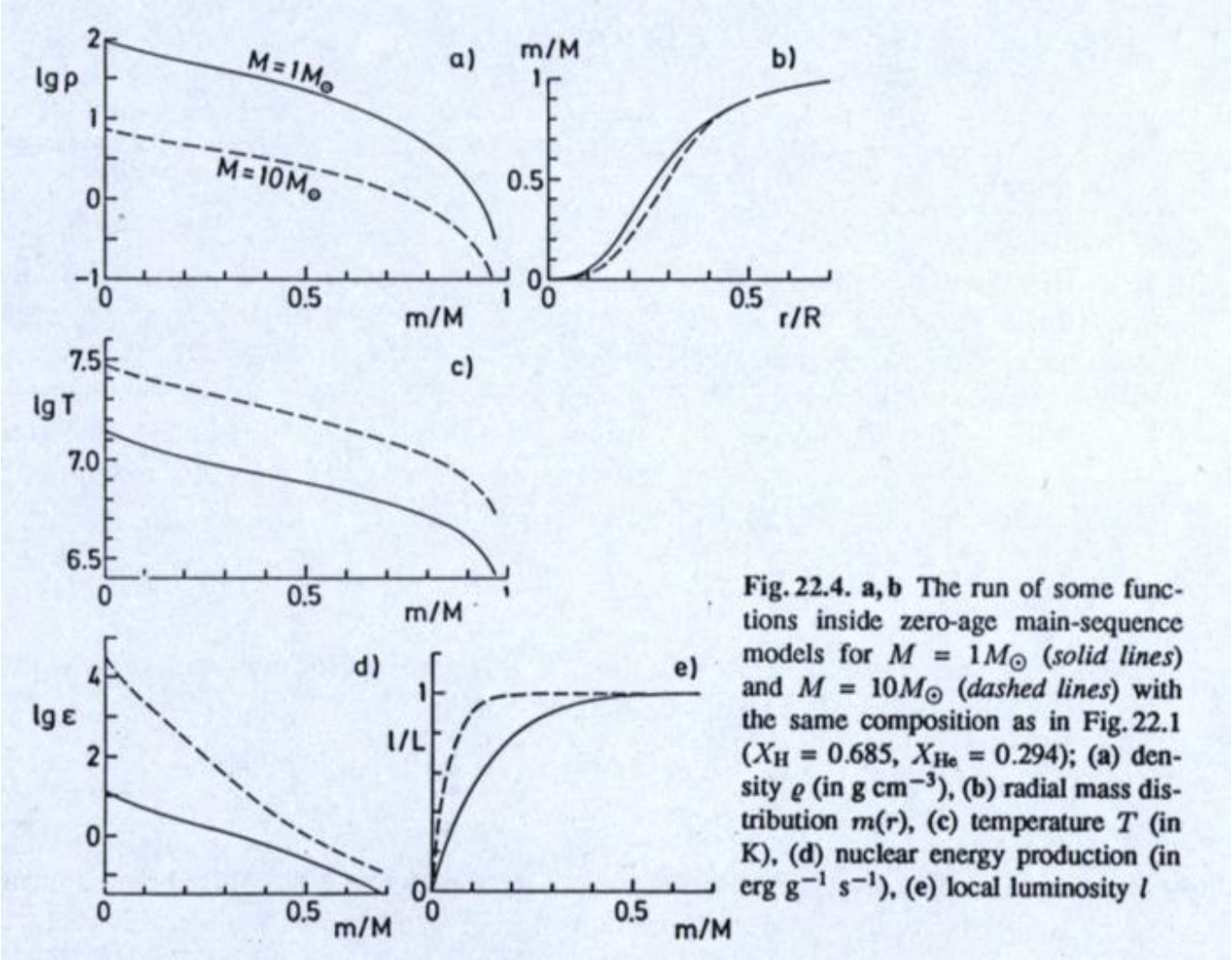


Fig. 22.4. **a, b** The run of some functions inside zero-age main-sequence models for $M = 1M_\odot$ (*solid lines*) and $M = 10M_\odot$ (*dashed lines*) with the same composition as in Fig. 22.1 ($X_H = 0.685$, $X_{He} = 0.294$); (**a**) density ϱ (in g cm^{-3}), (**b**) radial mass distribution $m(r)$, (**c**) temperature T (in K), (**d**) nuclear energy production (in erg g^{-1} s^{-1}), (**e**) local luminosity l

Kippenhahn & Weigert 1990

POLITROPY

$$P=K\rho^{\gamma}$$

$$\gamma = 1 + \frac{1}{n}$$

K - stała politropy
n - indeks politropy
γ - wykładnik politropy

Równanie politropy

(Lane'a-Emdena)

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right) = -\theta^n$$

$$\rho = \rho_\mathrm{c}\theta^n$$

$$r = \alpha\xi,$$

$$\alpha^2 = \frac{(n+1)K\rho_\mathrm{c}^{1/n\,-1}}{4\pi G}$$

Powierzchnia modelu jest wyznaczana przez pierwsze miejsce zerowe, θ=0

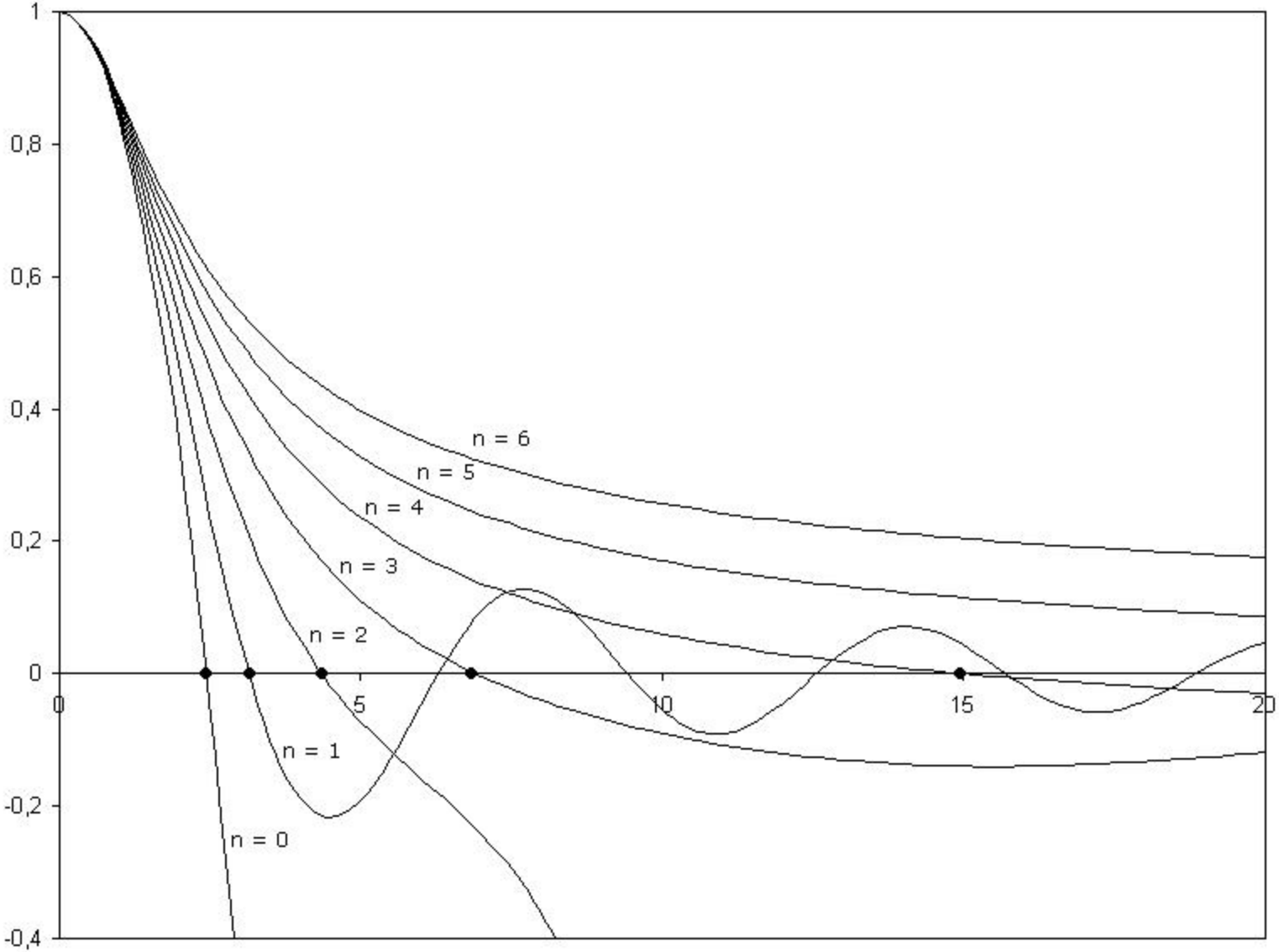
$\xi_1=\xi(0)$

Promień politropy

$$R = \left[\frac{(n+1)K\rho_{\rm c}^{1/n\,-1}}{4\pi G}\right]^{1/2} \xi_1$$

Masa

$$M = -4\pi \left[\frac{(n+1)K}{4\pi G}\right]^{3/2} \rho_{\rm c}^{(3-n)/2n} \left(\xi^2 \frac{{\rm d}\theta_n}{{\rm d}\xi}\right)_{\xi=\xi_1}$$


1
0,8
0,6
0,4
0,2
0
-0,2
-0,4
0
5
10
15
20
n = 6
n = 5
n = 4
n = 3
n = 2
n = 1
n = 0

Analityczne rozwiązania tylko dla:

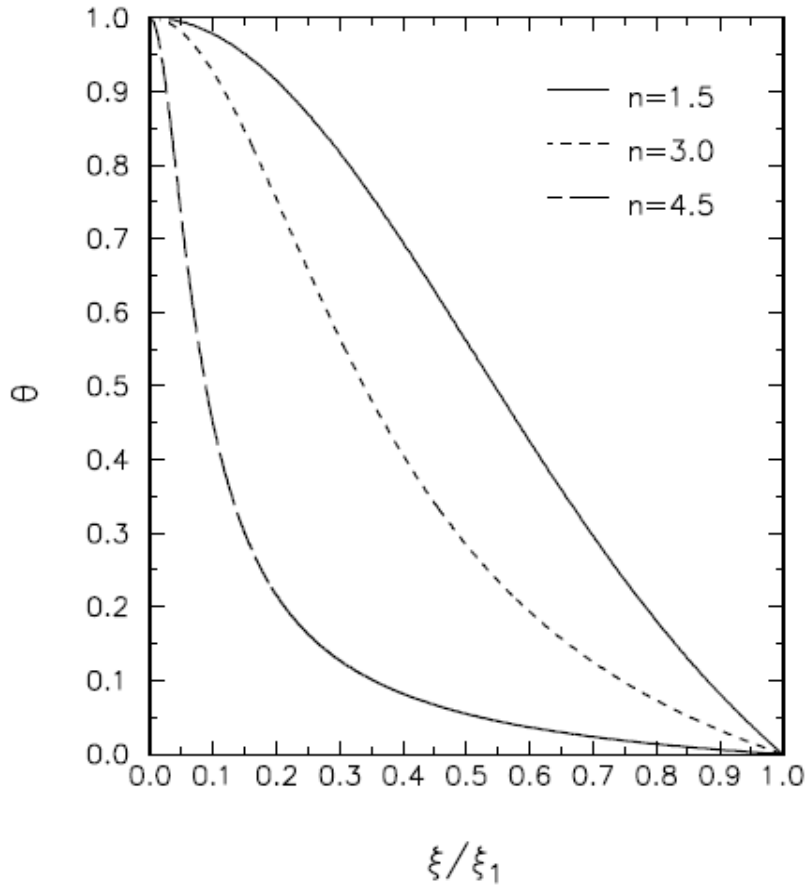
$$n = 0: \qquad \theta_0 = 1 - \frac{1}{6}\xi^2 \qquad \xi_1 = \sqrt{6}\,,$$

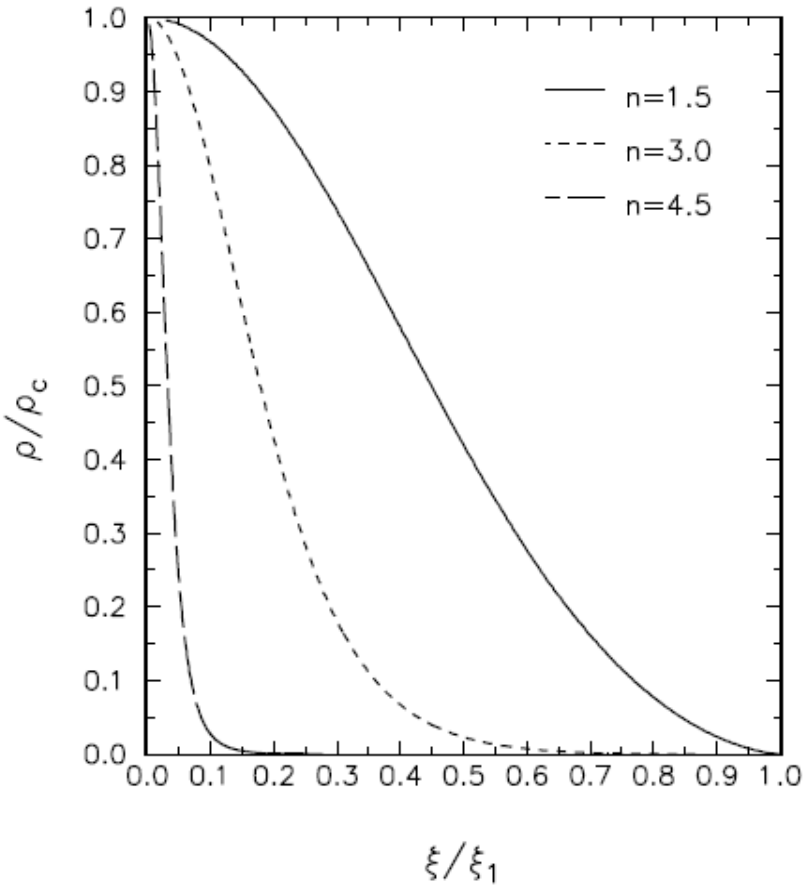
$$n = 1: \qquad \theta_1 = \frac{\sin\xi}{\xi} \qquad \xi_1 = \pi\,,$$

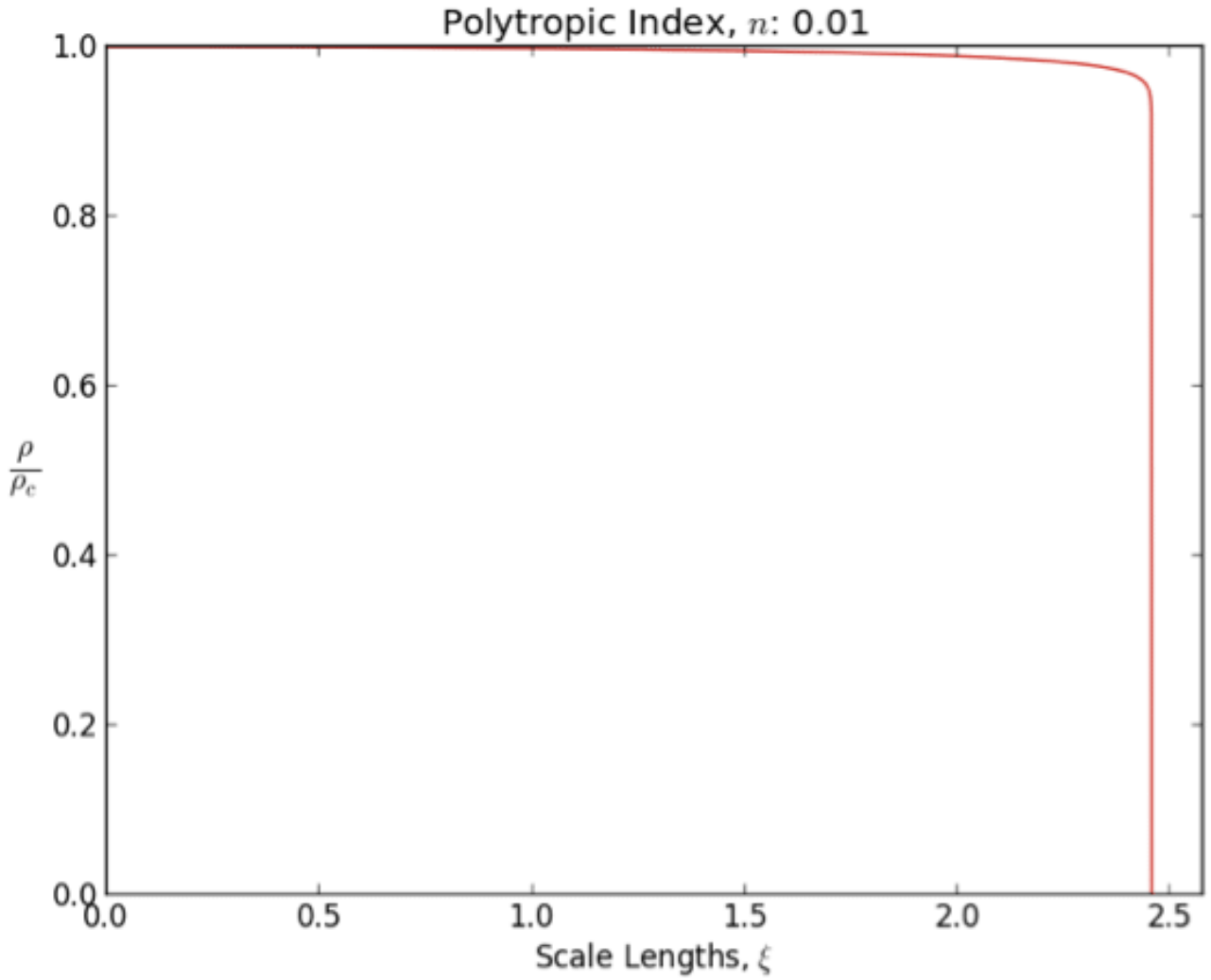
$$n = 5: \qquad \theta_5 = (1 + \frac{1}{3}\xi^2)^{-1/2} \qquad \xi_1 = \infty\,.$$

Parametry funkcji Lane'a-Emdena

n	ξ_1	$-\xi_1^2 \left(\frac{\mathrm{d}\theta_n}{\mathrm{d}\xi}\right)_{\xi=\xi_1}$	$\frac{\rho_c}{\overline{\rho}}$
0.	2.44949	4.89898	1.00000
0.50	2.75270	3.78865	1.83514
1.00	3.14159	3.14159	3.28987
1.50	3.65375	2.71406	5.99070
2.00	4.35287	2.41105	11.40254
2.50	5.35528	2.18720	23.40646
3.00	6.89685	2.01824	54.1825
3.25	8.01894	1.94980	88.1532
3.50	9.53581	1.89056	152.8837
4.00	14.97155	1.79723	622.408
4.50	31.83646	1.73780	6189.47
4.90	171.4335	1.72462	973806
5.0	∞	1.73205	∞

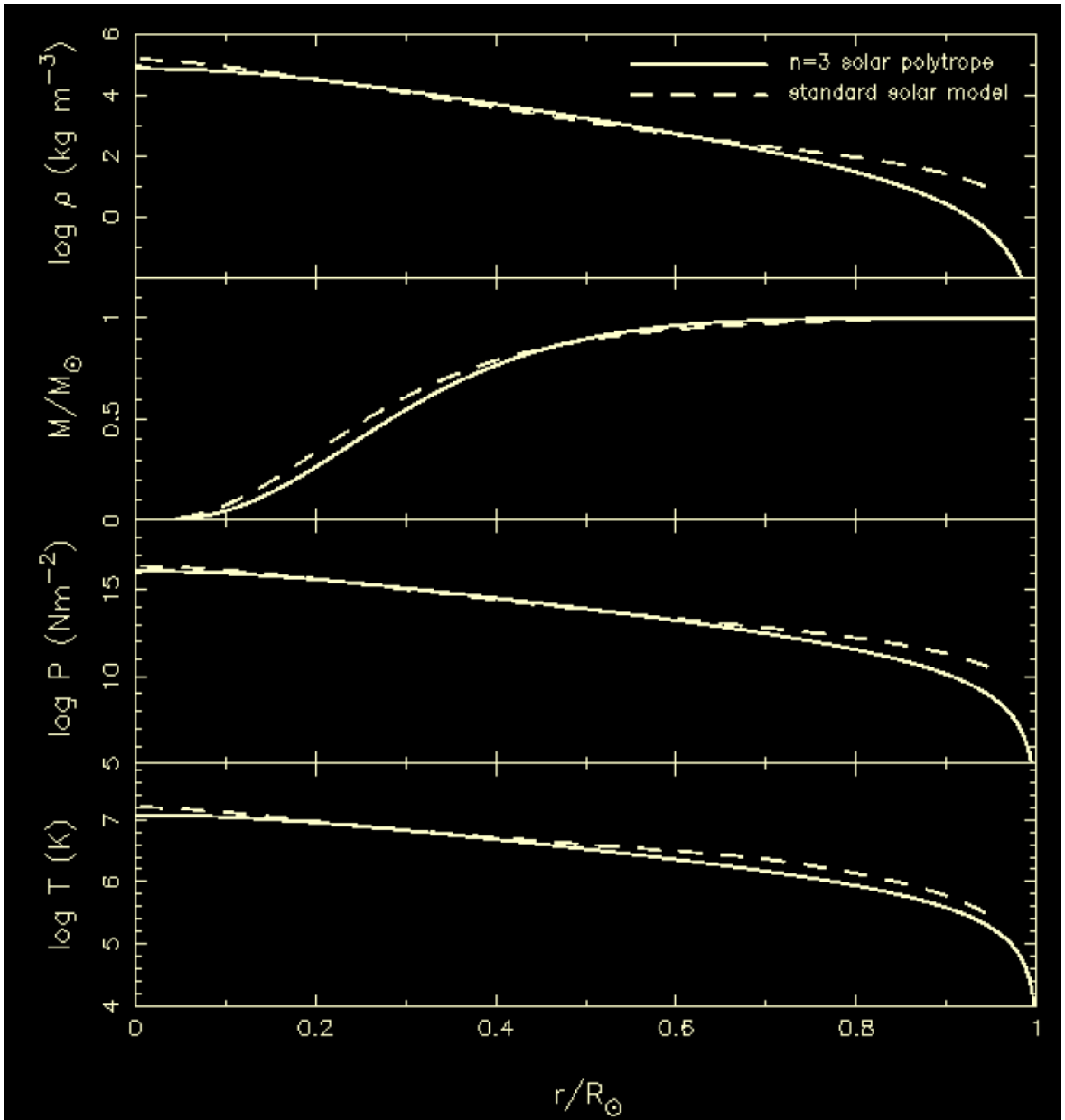

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
θ
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
ξ/ξ1
n=1.5
n=3.0
n=4.5


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
ρ/ρc
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
ξ/ξ1
n=1.5
n=3.0
n=4.5



Indeks politropy, *n*, rośnie to rośnie koncentracja masy w centralnych obszarach

Porównanie politropy *n*=3 z modelem standardowym Słońca



n	ξ_1	$-\xi_1^2 \left(\frac{\mathrm{d}\theta_n}{\mathrm{d}\xi}\right)_{\xi=\xi_1}$	$\frac{\rho_\mathrm{c}}{\overline{\rho}}$	N_n	W_n
0.	2.44949	4.89898	1.00000	$\cdots$	0.119366
0.50	2.75270	3.78865	1.83514	2.52360	0.212209
1.00	3.14159	3.14159	3.28987	0.63662	0.392699
1.50	3.65375	2.71406	5.99070	0.42422	0.770140
2.00	4.35287	2.41105	11.40254	0.36475	1.638183
2.50	5.35528	2.18720	23.40646	0.35150	3.909062
3.00	6.89685	2.01824	54.1825	0.36394	11.050679
3.25	8.01894	1.94980	88.1532	0.37898	20.36510
3.50	9.53581	1.89056	152.8837	0.40104	40.90982
4.00	14.97155	1.79723	622.408	0.47720	247.559
4.50	31.83646	1.73780	6189.47	0.65798	4921.84
4.90	171.4335	1.72462	973806	1.35323	3.916826×10^6
5.0	∞	1.73205	∞	∞	∞

Własności politropy
i indeksie n=3

ξ	θ_3	θ_3^3	θ_3^4	$-\frac{\mathrm{d}\theta_3}{\mathrm{d}\xi}$	q
0.	1.	1.	1.	0.	0.
0.25	0.98968	0.96936	0.95935	0.08179	0.00253
0.50	0.95984	0.88429	0.84878	0.15484	0.01918
0.75	0.91354	0.76240	0.69649	0.21263	0.05926
1.00	0.85506	0.62515	0.53454	0.25213	0.12493
1.25	0.78898	0.49113	0.38749	0.27367	0.21187
1.50	0.71950	0.37247	0.26800	0.27991	0.31206
1.75	0.64999	0.27461	0.17849	0.27460	0.41668
2.00	0.58285	0.19800	0.11541	0.26149	0.51826
2.25	0.51962	0.14030	0.07290	0.24380	0.61154
2.50	0.46113	0.09805	0.04521	0.22397	0.69358
2.75	0.40767	0.06775	0.02762	0.20369	0.76325
3.00	0.35923	0.04636	0.01665	0.18405	0.82074
3.25	0.31554	0.03142	0.009914	0.16567	0.86702
3.50	0.27626	0.02109	0.005825	0.14885	0.90344
3.75	0.24098	0.01399	0.003372	0.13369	0.93151
4.00	0.20928	0.009166	0.001918	0.12017	0.95266
4.25	0.18077	0.005907	0.001068	0.10819	0.96826
4.50	0.15507	0.003729	0.000578	0.09762	0.97946
4.75	0.13185	0.002292	0.000302	0.08831	0.98727
5.00	0.11082	0.001361	0.000151	0.08013	0.99252
5.25	0.09171	0.000771	0.000071	0.07292	0.99590
5.50	0.07429	0.000410	0.000030	0.06658	0.99795
5.75	0.05835	0.000199	0.000012	0.06099	0.99910
6.00	0.04374	0.000084	0.000004	0.05604	0.99967
6.25	0.03029	0.000028	0.000001	0.05166	0.99991
6.50	0.01787	0.000006	0.000000	0.04777	0.99999
6.75	0.00637	0.000000	0.000000	0.04430	1.00000
6.89685	0.00000	0.000000	0.000000	0.04243	1.00000

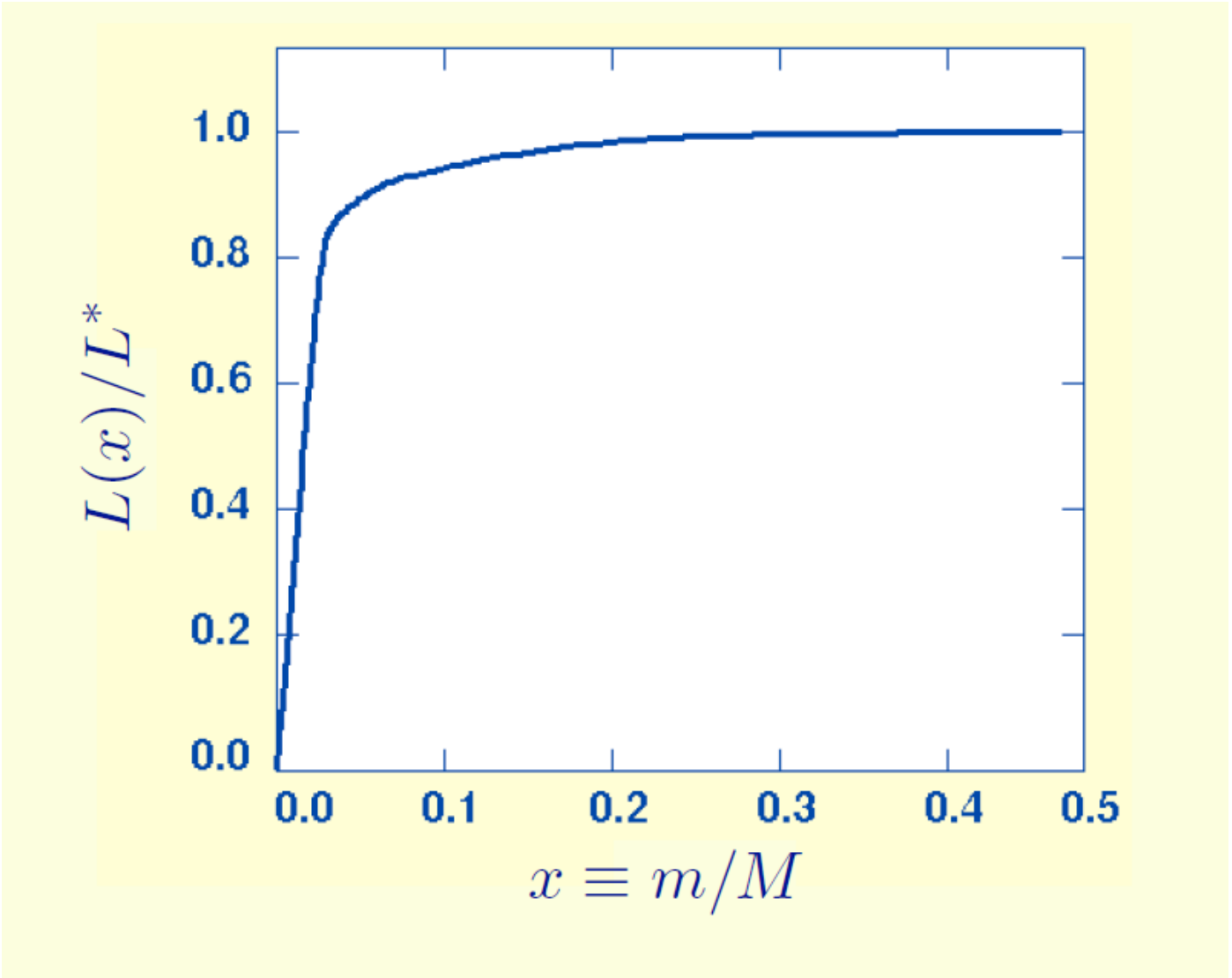
RELACJE HOMOLOGICZNE

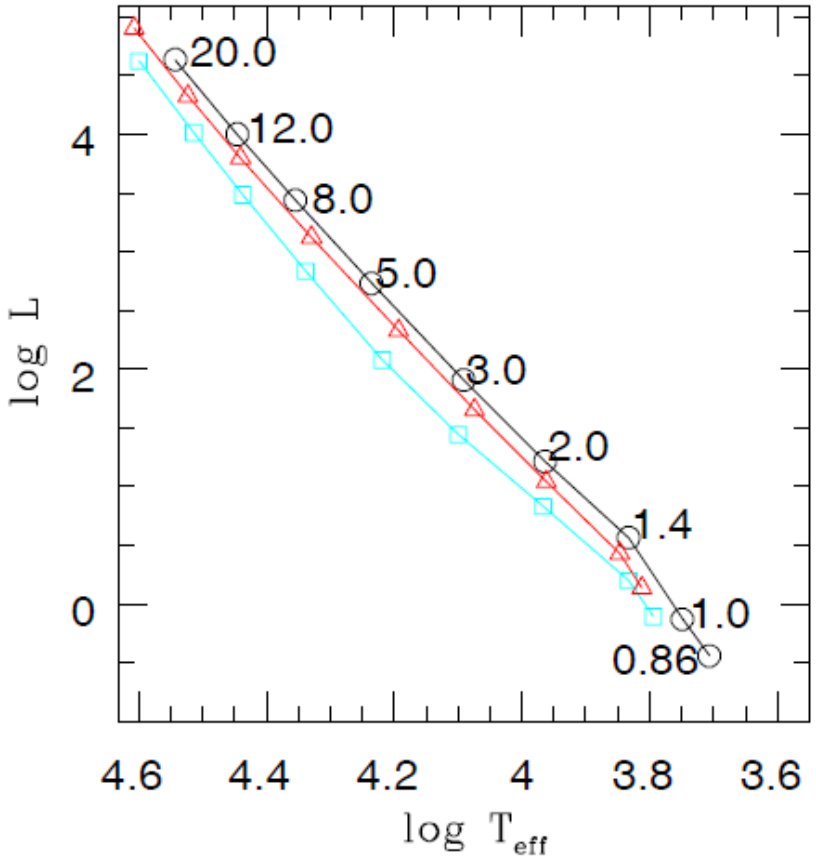
Homó logos (gr.) – zgodny, podobny

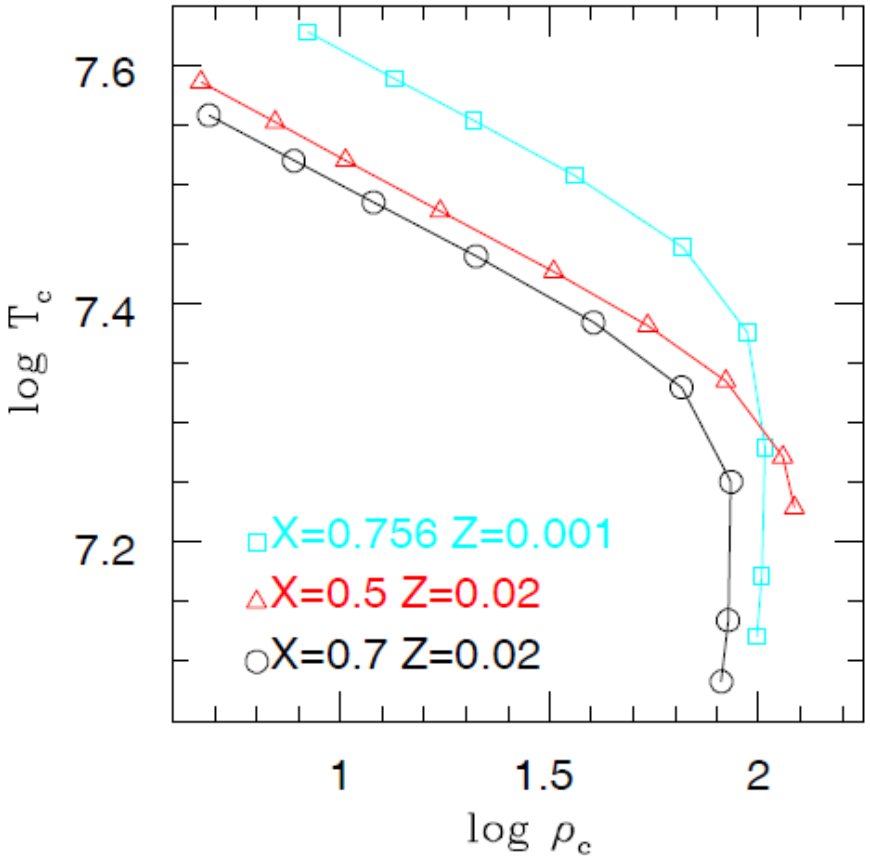
$$x = \frac{m_1}{M_1} = \frac{m_2}{M_2}$$

Dla każdego *x* zachodzi:

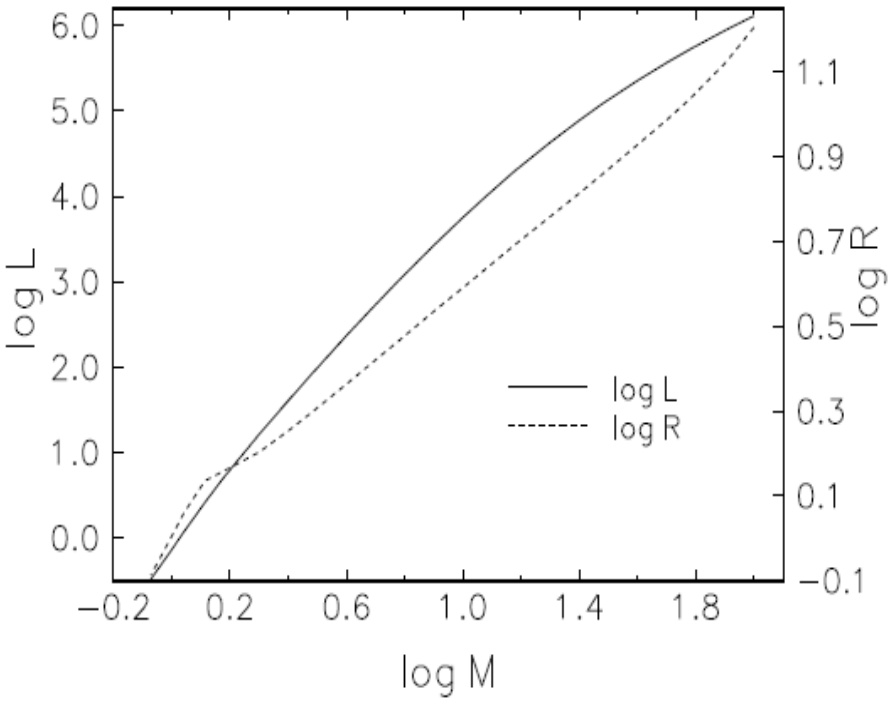
$$\frac{r_1(x)}{R_1} = \frac{r_2(x)}{R_2}$$

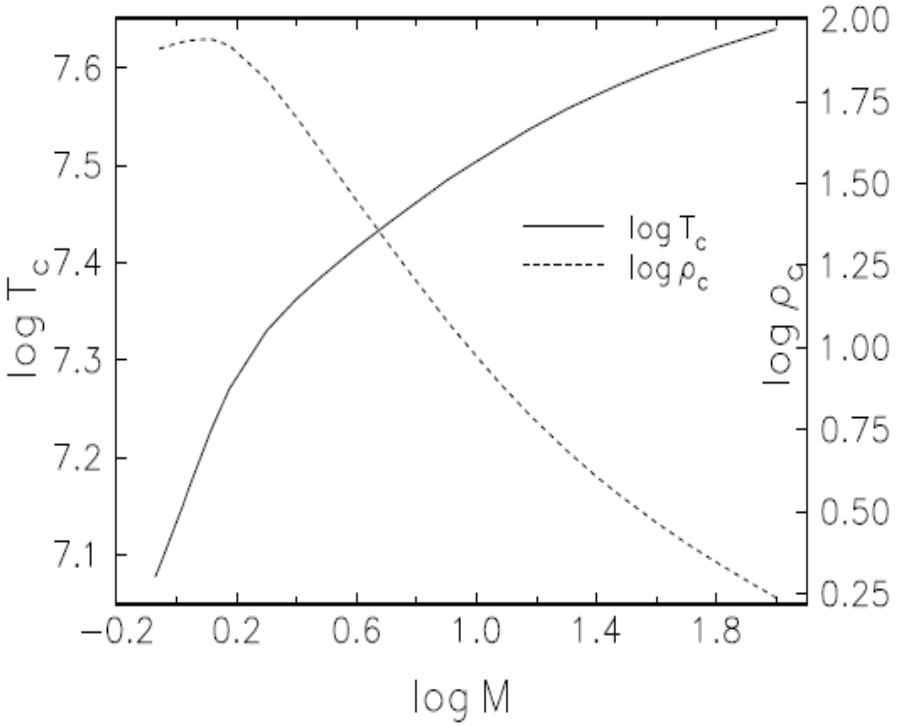

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
0.5
$L(x)/L^*$
$x \equiv m/M$





L i M w jednostkach słonecznych





L, M, R w jednostkach słonecznych